

Work, Energy and Power

Question1

A constant force of $(8\hat{i} - 2\hat{j} + 6\hat{k})\text{N}$ acting on a body of mass 2 kg displaces the body from $(2\hat{i} + 3\hat{j} - 4\hat{k})\text{m}$ to $(4\hat{i} - 3\hat{j} + 6\hat{k})\text{m}$. The work done in the process is

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Options:

A. 72 J

B. 88 J

C. 44 J

D. 36 J

Answer: B

Solution:

Work done by the force in displacing the body is given by,

$$W = \mathbf{F} \cdot \mathbf{s}$$

$$\text{Given, Force vector, } \mathbf{F} = (8\hat{i} - 2\hat{j} + 6\hat{k})$$

$$\text{Initial position vector, } r_1 = (2\hat{i} + 3\hat{j} - 4\hat{k})$$

$$\text{Final position vector, } r_2 = (4\hat{i} - 3\hat{j} + 6\hat{k})$$

The displacement vector s is given by

$$\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$$

$$\mathbf{s} = 2\hat{i} - 6\hat{j} + 10\hat{k}$$

Putting the value of \mathbf{F} and s in Eq. (i), we get



$$W = (8\hat{i} - 2\hat{j} + 6\hat{k}) \cdot (2\hat{i} - 6\hat{j} + 10\hat{k})$$

$$W = 16 + 12 + 60$$

$$W = 88 \text{ J}$$

Question2

The kinetic energy of a body of mass 4 kg moving with a velocity of $(2\hat{i} - 4\hat{j} - \hat{k})\text{ms}^{-1}$ is

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Options:

A. 84 J

B. 63 J

C. 42 J

D. 21 J

Answer: C

Solution:

Given:

Mass $m = 4 \text{ kg}$

Velocity $\mathbf{v} = (2\hat{i} - 4\hat{j} - \hat{k}) \text{ m/s}$

The kinetic energy, KE, of the body is calculated using the formula:

$$\text{KE} = \frac{1}{2}mv^2$$

To find v^2 :

$$v^2 = (2\hat{i} - 4\hat{j} - \hat{k}) \cdot (2\hat{i} - 4\hat{j} - \hat{k})$$

$$v = \sqrt{2^2 + (-4)^2 + (-1)^2}$$

$$v = \sqrt{4 + 16 + 1} = \sqrt{21} \text{ m/s}$$

Substituting the values back into the kinetic energy formula:



$$KE = \frac{1}{2} \times 4 \times (\sqrt{21})^2$$

$$= 2 \times 21$$

$$KE = 42 \text{ J}$$

Question3

A block kept on a frictionless horizontal surface is connected to one end of a horizontal spring of constant 100Nm^{-1} whose other end is fixed to a rigid vertical wall. Initially the block is at its equilibrium position. The block is pulled to a distance of 8 cm and released. The kinetic energy of the block when it is a distance of 3 cm from the mean position is

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Options:

A. 0.65 J

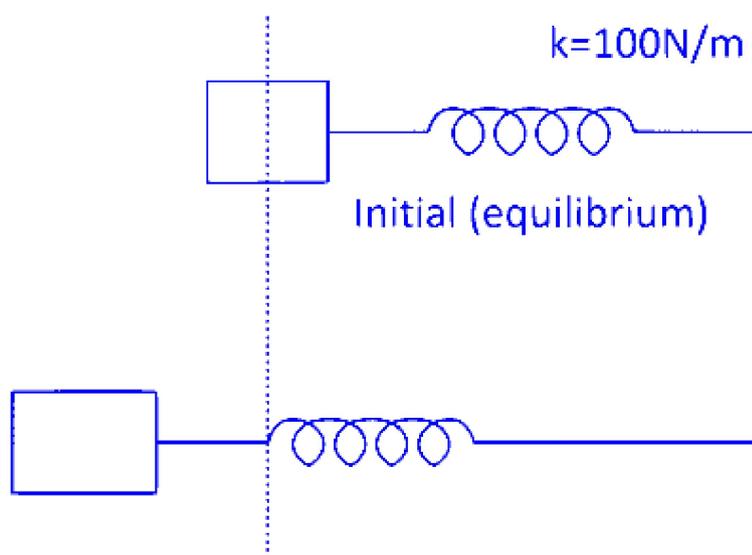
B. 0.325 J

C. 0.275 J

D. 0.55 J

Answer: C

Solution:



Spring constant, $k = 100 \text{ N/m}$

Spring stretched, $A = 8 \text{ cm}$

Position from mean where energy has $x = 3 \text{ cm}$ to be find.

Kinetic energy is given as

$$\begin{aligned} \text{KE} &= \frac{1}{2}k(A^2 - x^2) \\ &= \frac{1}{2} \times 100(8^2 - 3)^2 \times 10^{-4} \\ &= 50(64 - 9) \times 10^{-4} = 50 \times 55 \times 10^{-4} \end{aligned}$$

$$\text{KE} = 0.275 \text{ J}$$

Question4

The work done in blowing a soap bubble of volume V is W . The work done in blowing the bubble of volume $2V$ from the same soap solution is

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Options:

- A. $\frac{W}{2}$
- B. $\sqrt{2}W$
- C. $(2)^{\frac{1}{3}}W$
- D. $(4)^{\frac{1}{3}}W$

Answer: D

Solution:

The work done in blowing a soap bubble is given by the formula $W = T \cdot \Delta A$, where T represents the surface tension and ΔA indicates the change in surface area.

The volume of a sphere (representing the soap bubble) is given by:

$$V = \frac{4}{3}\pi r^3$$



where r is the radius of the soap bubble.

The surface area of the sphere is:

$$A = 4\pi r^2$$

From the above formulas, we can deduce that:

$$A \propto V^{\frac{2}{3}}$$

Therefore, the change in area, ΔA , is proportional to $(\Delta V)^{\frac{2}{3}}$.

To find the ratio of work done in expanding the bubble, we use:

$$\frac{W_1}{W_2} = \frac{\Delta A_1}{\Delta A_2} = \left(\frac{\Delta V_1}{\Delta V_2}\right)^{\frac{2}{3}}$$

For a bubble expanding from volume V to volume $2V$:

$$W_2 = W_1 \left(\frac{2V}{V}\right)^{\frac{2}{3}}$$

Substituting $W_1 = W$ (the original work done for volume V):

$$W_2 = W[2]^{\frac{2}{3}}$$

Which simplifies to:

$$W_2 = W(4)^{\frac{1}{3}}$$

Question5

A body thrown vertically upwards from the ground reaches a maximum height h . The ratio of the kinetic and potential energies of the body at a height 40% of h from the ground is

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Options:

A. 2 : 3

B. 3 : 2

C. 1 : 1

D. 4 : 9

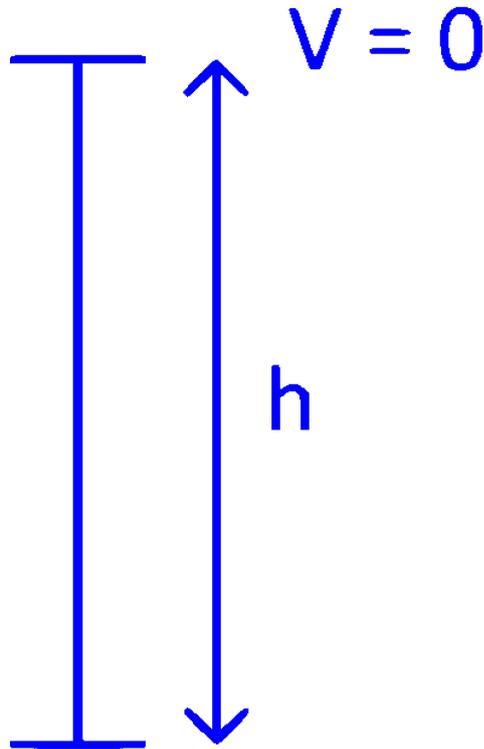
Answer: B



Solution:

Body mass = m

At maximum height h



According to conservation of energy,

Total Energy (TE) = Potential Energy (PE)

Kinetic Energy (KE)

at maximum height velocity = 0

So,

$$TE = PE = mgh$$

Now, at 40% of maximum height = $\frac{2}{5}h$

$$(PE)_{40\%} = mg \left(\frac{2}{5}h \right)$$

Applying conservation of energy equation to find KE at 40% maximum height

$$\begin{aligned}(\text{KE})_{40\%} &= (\text{TE}) - (\text{PE})_{40\%} \\ &= mgh - mg \left(\frac{2}{5}h \right) \\ &= \frac{3mgh}{5}\end{aligned}$$

Taking the ratio of $(\text{KE})_{40\%}$ and $(\text{PE})_{40\%}$

$$\frac{(\text{KE})_{40\%}}{(\text{PE})_{40\%}} = \frac{\frac{3mgh}{5}}{\frac{2mgh}{5}} = \frac{3}{2} = 3 : 2$$

Question6

A block of mass m with an initial kinetic energy E moves up an inclined plane of inclination θ . If μ is the coefficient of friction between the plane and the body, the work done against friction before coming to rest is

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Options:

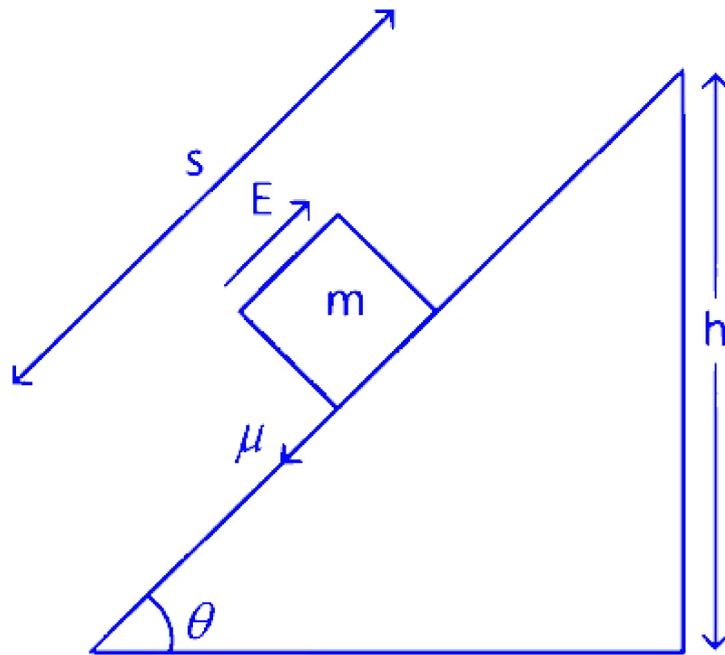
- A. $\mu E \cos \theta$
- B. $\frac{\mu E \cos \theta}{\sin \theta - \mu \cos \theta}$
- C. $\frac{E \mu \cos \theta}{\cos \theta + \sin \theta}$
- D. $\frac{\mu E \cos \theta}{\sin \theta + \mu \cos \theta}$

Answer: D

Solution:

Let the body slide a distance s along the incline before coming to rest.

So, it ascends by a vertical height $h = s \sin \theta$



From work-energy theorem,

$$\Delta W_{ff} + \Delta W_{mg} = \Delta KE$$

$$\Rightarrow -\mu mg(\cos \theta)s - mgs \sin \theta = 0 - E$$

$$\Rightarrow \mu mgs \cos \theta + mgs \sin \theta = E$$

$$\Rightarrow s = \frac{E}{mg(\mu \cos \theta + \sin \theta)}$$

So work done by friction,

$$\Delta W_{ff} = -\mu mgs \cos \theta = \frac{-\mu E \cos \theta}{\mu \cos \theta + \sin \theta}$$

-ve sign is for direction.

Question7

A man of mass 80 kg goes to the market on a scooter of mass 100 kg with certain speed. On application of brakes, the stopping distance is s_1 . The man returns home on the same scooter, with the same speed with a 60 kg bag of rice. If s_2 is the new stopping distance when the brakes are applied with the same force, then

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Options:

A. $7s_1 = 4s_2$

B. $2s_1 = s_2$

C. $3s_1 = 4s_2$

D. $4s_1 = 3s_2$

Answer: D

Solution:

The stopping distance of a moving object depends on its mass, velocity, and the force applied to stop it. The formula for stopping distance can be obtained from the work-energy principle:

$$F \cdot s = \frac{1}{2}mv^2$$

where:

F is the force applied by the brakes,

s is the stopping distance,

m is the mass of the moving object,

v is the velocity of the object.

Assuming the brakes apply a constant stopping force in both scenarios, we compare the initial and new stopping scenarios.

Initial Scenario:

Total mass $m_1 = 80 \text{ kg (mass of man)} + 100 \text{ kg (mass of scooter)} = 180 \text{ kg}$

Stopping distance: s_1

New Scenario (with bag of rice):

New total mass $m_2 = 80 \text{ kg (mass of man)} + 100 \text{ kg (mass of scooter)} + 60 \text{ kg (mass of rice)} = 240 \text{ kg}$

New stopping distance: s_2

Analysis:

Using the work-energy principle for both scenarios and assuming velocity v and force F remain constant:

Initial condition:

$$F \cdot s_1 = \frac{1}{2} \times 180 \times v^2$$

New condition:

$$F \cdot s_2 = \frac{1}{2} \times 240 \times v^2$$

Dividing the second equation by the first:



$$\frac{s_2}{s_1} = \frac{\frac{1}{2} \times 240 \times v^2}{\frac{1}{2} \times 180 \times v^2} = \frac{240}{180} = \frac{4}{3}$$

Rearranging gives:

$$s_2 = \frac{4}{3} s_1$$

or equivalently:

$$3s_2 = 4s_1$$

Thus, the correct relationship is given by:

Option D: $4s_1 = 3s_2$

Question8

A block of mass 0.5 kg is at rest on a horizontal table. The coefficient of kinetic friction between the table and the block is 0.2 . If a horizontal force of 5 N is applied on the block, the kinetic energy of the block in a time of 4 s is (Acceleration due to gravity = 10 ms^{-2})

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Options:

A. 64 J

B. 128 J

C. 256 J

D. 512 J

Answer: C

Solution:

Given, mass of block = 0.5 kg

Kinetic friction, $\mu_k = 0.2$

Horizontal force, $F = 5 \text{ N}$

$t = 4 \text{ s}$

Frictional force,



$$f_{\text{friction}} = \mu_k \times m \times g$$

$$f_{\text{friction}} = 1 \text{ N}$$

Net force

$$F_{\text{net}} = F_{\text{applied}} - f_{\text{friction}}$$

$$F_{\text{net}} = 4 \text{ N}$$

Acceleration,

$$a = \frac{F_{\text{net}}}{m}$$

$$\Rightarrow a = 8 \text{ m/s}^2$$

Velocity, $v = u + a \times t$

$$\Rightarrow v = 0 + 8 \times 4 = 32 \text{ m/s}$$

Kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 0.5 \times 1024$$

$$= 256 \text{ J}$$

Kinetic energy at 4 s is 256 J .

Question9

The work done in increasing the diameter of a soap bubble from 2 cm to 4 cm is (Surface tension of soap solution = $3.5 \times 10^{-2} \text{Nm}^{-1}$)

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Options:

A. $528 \times 10^{-6} \text{ J}$

B. $264 \times 10^{-6} \text{ J}$

C. $132 \times 10^{-6} \text{ J}$

D. $178 \times 10^{-6} \text{ J}$



Answer: B

Solution:

Given, initial diameter = 2 cm

Initial radius, $r_1 = 0.01$ m

Final diameter = 4 cm

Final radius, $r_2 = 0.02$ m

Surface Tension, $T = 3.5 \times 10^{-2} \text{Nm}^{-1}$

Work done, $W = T \times \Delta A$... (i)

where, ΔA is change in surface area

$$\Delta A = 2 \times 4\pi (r_2^2 - r_1^2) \quad \dots \text{ (ii)}$$

$$= 2 \times 4\pi [(0.02)^2 - (0.01)^2]$$

$$\Delta A = 8\pi(0.0003)$$

$$\Delta A = 2.4\pi \times 10^{-3} \text{ m}^2 \quad \dots \text{ (iii)}$$

Putting value of Eq. (iii) in Eq. (i)

Thus,

$$W = 3.5 \times 10^{-2} \times 2.4 \times \pi \times 10^{-3} \text{ J}$$

$$W = 8.4\pi \times 10^{-5}$$

$$W = 8.4 \times 3.14 \times 10^{-5} = 26.3 \times 10^{-5}$$

$$W = 263 \times 10^{-6} \text{ J}$$

Question10

While a person climbs stairs, the gravitational potential energy of the person increases. The source of this energy is

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Options:



- A. work done by normal force from the steps
- B. work done by frictional force from the steps
- C. work done by air resistance
- D. work done by internal forces within the person's body

Answer: D

Solution:

When a person climbs stairs, their gravitational potential energy increases. But where does this energy come from? The source is the work done by internal forces within the person's body.

Let's break it down:

Consider a person starting at the ground level, where the initial height $h = 0$.

As the person climbs and reaches a height h , their potential energy increases by:

$$mgh$$

where m is the mass of the person, g is the acceleration due to gravity, and h is the height climbed.

The change in potential energy is mathematically expressed as:

$$PE_f - PE_i = \int dW$$

where dW represents the work done.

In this scenario, the increase in gravitational potential energy results from the work done by the internal forces within the person's body, such as the muscles exerting force to climb the stairs.

Question11

Two bodies of masses of 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is

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Options:

- A. 4 : 1
- B. $\sqrt{2}$: 1
- C. 1 : 2

D. 1 : 16

Answer: C

Solution:

Given two bodies with masses $m_1 = 1$ g and $m_2 = 4$ g, both are moving with equal kinetic energies. To find the ratio of their linear momenta, we can use the formula for linear momentum, which is related to kinetic energy as follows:

$$p = \sqrt{2m(\text{KE})}$$

For the first body:

$$p_1 = \sqrt{2 \cdot 1 \cdot \text{KE}}$$

For the second body:

$$p_2 = \sqrt{2 \cdot 4 \cdot \text{KE}}$$

To find the ratio of the magnitudes of their linear momenta, divide the expressions of p_1 and p_2 :

$$\frac{p_1}{p_2} = \frac{\sqrt{2 \cdot \text{KE}}}{\sqrt{8 \cdot \text{KE}}}$$

Since the kinetic energies are equal, they cancel out in the equation:

$$\frac{p_1}{p_2} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 1 : 2$$

Thus, the ratio of the magnitudes of their linear momenta is 1 : 2.

Question12

The displacement s of a body of mass 3 kg under the action of a force is given by $s = \frac{t^3}{3}$, where s is in metres and t is in seconds. The work done by the force in the first two seconds is

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Options:

A. 32 J

B. 3.8 J

C. 5.2 J



D. 24 J

Answer: D

Solution:

Given that the displacement s of a body of mass 3 kg under the action of a force is defined by the equation $s = \frac{t^3}{3}$, where s is in meters and t is in seconds, we need to find the work done by the force in the first two seconds.

Here is how we can calculate it:

Displacement Equation:

$$s = \frac{t^3}{3}$$

Calculate Velocity:

Velocity v is the derivative of displacement with respect to time:

$$v = \frac{ds}{dt} = t^2$$

Calculate Acceleration:

Acceleration a is the derivative of velocity with respect to time:

$$a = \frac{dv}{dt} = 2t$$

Force Equation:

Force F is mass times acceleration:

$$F = m \cdot a = 3 \times (2t) = 6t$$

Work Done Calculation:

Work W is the integral of F with respect to displacement ds :

$$W = \int F \cdot ds = \int_0^2 6t \cdot t^2 dt = \int_0^2 6t^3 dt$$

Evaluate the Integral:

$$\begin{aligned} W &= 6 \int_0^2 t^3 dt \\ &= 6 \left[\frac{t^4}{4} \right]_0^2 \\ &= 6 \left[\frac{16}{4} - 0 \right] \\ &= 6 \times 4 \\ &= 24 \text{ Joules} \end{aligned}$$

Hence, the work done by the force in the first two seconds is 24 Joules.

